# NAG Toolbox for MATLAB

# f08qy

## 1 Purpose

f08qy estimates condition numbers for specified eigenvalues and/or right eigenvectors of a complex upper triangular matrix.

## 2 Syntax

```
[s, sep, m, info] = f08qy(job, howmny, select, t, vl, vr, mm, 'n', n)
```

## 3 Description

f08qy estimates condition numbers for specified eigenvalues and/or right eigenvectors of a complex upper triangular matrix $T$. These are the same as the condition numbers of the eigenvalues and right eigenvectors of an original matrix $A = ZTZ^{\mathrm{H}}$ (with unitary $Z$), from which $T$ may have been derived.

f08qy computes the reciprocal of the condition number of an eigenvalue $\lambda_i$ as

$$s_i = \frac{\left|v^{\mathrm{H}}u\right|}{\|u\|_E\|v\|_E},$$

where $u$ and $v$ are the right and left eigenvectors of $T$, respectively, corresponding to $\lambda_i$. This reciprocal condition number always lies between zero (i.e., ill-conditioned) and one (i.e., well-conditioned).

An approximate error estimate for a computed eigenvalue $\lambda_i$ is then given by

$$\frac{\epsilon\|T\|}{s_i},$$

where $\epsilon$ is the ***machine precision***.

To estimate the reciprocal of the condition number of the right eigenvector corresponding to $\lambda_i$, the function first calls f08qt to reorder the eigenvalues so that $\lambda_i$ is in the leading position:

$$T = Q\begin{pmatrix} \lambda_i & c^{\mathrm{H}} \\ 0 & T_{22} \end{pmatrix}Q^{\mathrm{H}}.$$

The reciprocal condition number of the eigenvector is then estimated as $sep_i$, the smallest singular value of the matrix $(T_{22} - \lambda_i I)$. This number ranges from zero (i.e., ill-conditioned) to very large (i.e., well-conditioned).

An approximate error estimate for a computed right eigenvector $u$ corresponding to $\lambda_i$ is then given by

$$\frac{\epsilon\|T\|}{sep_i}.$$

## 4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **job – string**

Indicates whether condition numbers are required for eigenvalues and/or eigenvectors.

**job** = 'E'

Condition numbers for eigenvalues only are computed.

**job** = 'V'

Condition numbers for eigenvectors only are computed.

**job** = 'B'

Condition numbers for both eigenvalues and eigenvectors are computed.

*Constraint*: **job** = 'E', 'V' or 'B'.

2: **howmny – string**

Indicates how many condition numbers are to be computed.

**howmny** = 'A'

Condition numbers for all eigenpairs are computed.

**howmny** = 'S'

Condition numbers for selected eigenpairs (as specified by **select**) are computed.

*Constraint*: **howmny** = 'A' or 'S'.

3: **select**$(*)$ **– logical array**

**Note**: the dimension of the array **select** must be at least $\max(1, \mathbf{n})$ if **howmny** = 'S', and at least 1 otherwise.

Specifies the eigenpairs for which condition numbers are to be computed if **howmny** = 'S'. To select condition numbers for the eigenpair corresponding to the eigenvalue $\lambda_j$, **select**$(j)$ must be set to **true**.

If **howmny** = 'A', **select** is not referenced.

4: **t**(**ldt**,$*$) **– complex array**

The first dimension of the array **t** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The $n$ by $n$ upper triangular matrix $T$, as returned by f08ps.

5: **vl**(**ldvl**,$*$) **– complex array**

The first dimension, **ldvl**, of the array **vl** must satisfy

if **job** = 'E' or 'B', **ldvl** $\geq \max(1, \mathbf{n})$;
if **job** = 'V', **ldvl** $\geq 1$.

The second dimension of the array must be at least $\max(1, \mathbf{mm})$ if **job** = 'E' or 'B' and at least 1 if **job** = 'V'

If **job** = 'E' or 'B', **vl** must contain the left eigenvectors of $T$ (or of any matrix $QTQ^{\mathrm{H}}$ with $Q$ unitary) corresponding to the eigenpairs specified by **howmny** and **select**. The eigenvectors **must** be stored in consecutive columns of **vl**, as returned by f08qx or f08px.

If **job** = 'V', **vl** is not referenced.

6: **vr**(**ldvr**,$*$) **– complex array**

The first dimension, **ldvr**, of the array **vr** must satisfy

if **job** = 'E' or 'B', **ldvr** $\geq \max(1, \mathbf{n})$;
if **job** = 'V', **ldvr** $\geq 1$.

The second dimension of the array must be at least $\max(1, \mathbf{mm})$ if **job** = 'E' or 'B' and at least 1 if **job** = 'V'

If **job** = 'E' or 'B', **vr** must contain the right eigenvectors of $T$ (or of any matrix $QTQ^{\mathrm{H}}$ with $Q$ unitary) corresponding to the eigenpairs specified by **howmny** and **select**. The eigenvectors **must** be stored in consecutive columns of **vr**, as returned by f08qx or f08px.

If **job** = 'V', **vr** is not referenced.

7: **mm – int32 scalar**

The number of elements in the arrays **s** and **sep**, and the number of columns in the arrays **vl** and **vr** (if used). The precise number required, $m$, is $n$ if **howmny** = 'A'; if **howmny** = 'S', $m$ is the number of selected eigenpairs (see **select**), in which case $0 \leq m \leq n$.

*Constraint*: **mm** $\geq m$.

## 5.2 Optional Input Parameters

1: **n – int32 scalar**

*Default*: The second dimension of the array **t**.

$n$, the order of the matrix $T$.

*Constraint*: **n** $\geq 0$.

## 5.3 Input Parameters Omitted from the MATLAB Interface

ldt, ldvl, ldvr, work, ldwork, rwork

## 5.4 Output Parameters

1: **s**$(*)$ **– double array**

**Note**: the dimension of the array **s** must be at least $\max(1, \mathbf{mm})$ if **job** = 'E' or 'B' and at least 1 if **job** = 'V'.

The reciprocal condition numbers of the selected eigenvalues if **job** = 'E' or 'B', stored in consecutive elements of the array. Thus **s**($j$), **sep**($j$) and the $j$th columns of **vl** and **vr** all correspond to the same eigenpair (but not in general the $j$th eigenpair unless all eigenpairs have been selected).

**s** is not referenced if **job** = 'V'.

2: **sep**$(*)$ **– double array**

**Note**: the dimension of the array **sep** must be at least $\max(1, \mathbf{mm})$ if **job** = 'V' or 'B' and at least 1 if **job** = 'E'.

The estimated reciprocal condition numbers of the selected right eigenvectors if **job** = 'V' or 'B', stored in consecutive elements of the array.

If **job** = 'E', **sep** is not referenced i.

3: **m – int32 scalar**

$m$, the number of selected eigenpairs. If **howmny** = 'A', **m** is set to $n$.

4: **info – int32 scalar**

**info** = 0 unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

**info** $= -i$

If **info** $= -i$, parameter $i$ had an illegal value on entry. The parameters are numbered as follows:

1: **job**, 2: **howmny**, 3: **select**, 4: **n**, 5: **t**, 6: **ldt**, 7: **vl**, 8: **ldvl**, 9: **vr**, 10: **ldvr**, 11: **s**, 12: **sep**, 13: **mm**, 14: **m**, 15: **work**, 16: **ldwork**, 17: **rwork**, 18: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

## 7 Accuracy

The computed values $sep_i$ may over estimate the true value, but seldom by a factor of more than 3.

## 8 Further Comments

The real analogue of this function is f08ql.

## 9 Example

```
job = 'Both';
howmny = 'All';
select = [false];
t = [complex(-6.0004, -6.9999), complex(0.3637, -0.3656), complex(-0.188,
...
     +0.4787), complex(0.8784999999999999, -0.2539);
       complex(0, +0), complex(-5, +2.006), complex(-0.0307, -0.7217),
complex(-0.229, +0.1313);
          complex(0, +0), complex(0, +0), complex(7.9982, -0.9964),
complex(0.9357, +0.5359);
       complex(0, +0), complex(0, +0), complex(0, +0), complex(3.0023, -
3.9998)];
vl = [complex(1, +0), complex(0, +0), complex(0, +0), complex(0, +0);
      complex(0.03566943515948521, -0.04434689513913646), complex(1, +0),
complex(0, +0), complex(0, +0);
         complex(-0.002207376074206878, +0.03131341253433394), complex(-
0.009933197113414047, ...
     -0.05322864465746685), complex(1, +0), complex(0, +0);
                  complex(-0.07824388063236407, -0.05827078558456029),
complex(0.03187495330451339, ...
                  -0.001955906687244087), complex(0.1849408889864736,
+0.0039135022682549), complex(1, +0)];
vr     =     [complex(1,     +0),     complex(-0.03566943515948521,     -
0.04434689513913646), ...
                complex(-0.0005074605791794694, +0.03277154177278579),
complex(0.0792597025312869, -0.06285024830308554);
        complex(0, +0), complex(1, +0), complex(0.009933197113414047, -
0.05322864465746685), ...
     complex(-0.03350369718754292, +0.00792711976468731);
          complex(0, +0), complex(0, +0), complex(1, +0), complex(-
0.1849408889864736, +0.0039135022682549);
     complex(0, +0), complex(0, +0), complex(0, +0), complex(1, +0)];
mm = int32(4);
[s, sep, m, info] = f08qy(job, howmny, select, t, vl, vr, mm)
```

```
s =
    0.9932
    0.9964
    0.9814
    0.9779
sep =
```

```
    8.4012
    8.0215
    5.8292
    5.8292
m =
           4
info =
           0
```